FLORIDA STATE UNIVERSITY

DEPARTMENT OF METEOROLOGY

TECHNICAL REPORT NO. 6

A Study of Small-Scale Atmospheric Motion, 2: Divergence and Vorticity

By Holbrook Landers

PREPARED UNDER PROJECT NR-082-071, CONTRACT NONR-1600(00)

WITH THE OFFICE OF NAVAL RESEARCH

15 APRIL 1956

Department of Meteorology
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Tallahassee, Florida

Technical Report No. 6
Contract No. Nonr-1600(00)

A STUDY OF SMALL-SCALE ATMOSPHERIC MOTION, 2: DIVERGENCE AND VORTICITY

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This document, together with Technical Report No. 2 under Contract Nonr-1600(00), completes the report on one phase of research which has been made possible through partial support by the Geophysics Branch, Office of Naval Research.

Publication approved: Werner A. Baum

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Principal Investigator

Abstract

Divergence and vorticity are computed for a small area of about 1/2 square mile from the changes in area and orientation of a triangle formed by three pilot balloons as they ascend. Mean local, seasonal ascention rates were previously determined at 0300 Z and 1500 Z for the purpose of making these divergence and vorticity computations. The magnitudes and vertical distributions of the divergence and vorticity are discussed. Levels of least divergence are located at about 6000 ft and 13,000 ft, showing close agreement with an earlier study which dealt with a scale of motion compatible with the normal upper wind network. In one triple-balloon run there is strong evidence that a narrow zone (no more than 3000 ft wide) may exist in a horizontal plane across which the wind direction changes about 130°.

1. Introduction

In Part 1 of this study [1] mean local ascentional rates were determined for the thirty-gram balloon at the times 0300 Z (night) and 1500 Z (day) at Tallahassee, Florida in the summer of 1954. The details are described in [1]; suffice it to say here that both the day and night mean rates were found to be significantly greater than the normally assumed rate after the first 5 or 6 minutes of ascent (see fig. 2 in [1]). The mean local rates were greater than the assumed rate by as much as 9 per cent and the systematic nature of the difference caused the height of the balloon at higher levels to be considerably different from that normally assumed. For example, at the end of thirty minutes the balloon would be found at 18,012 ft, 18,602 ft and 19,064 ft using the assumed rate, the mean local day rate and the mean local night rate, respectively.

For the second phase of the study it is important to know as accurately as possible the height of the balloons at any given time and the rate of ascent and changes in this rate. This was the reason for determining mean local day and night rates and these mean rates are used in determining the divergence and vorticity.

2. The method

A triangle of theodolite stations was set up (see [1]) with the legs of the triangle being roughly one mile in length. Three balloons released simultaneously (one at each site) describe a triangular area at each minute reading. The changes in size and orientation of this area can then be expressed as the divergence and vorticity of the flow in this small region.

For the horizontal velocity divergence we can write,

$$\nabla_H \cdot V_H = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{1}{A}\frac{dA}{dt} \qquad (1)$$

where x and y are the sides of area A, t represents time and u and v are the wind speed components in the east (x) and north (y) directions. At time zero, A is the area of the fixed triangle of theodolite stations.

The area at the end of each minute interval can be obtained from the determinant

$$A_t = 1/2 \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \qquad (2)$$

Fig. 1 shows the release sites 1, 2, and 3 and the orientation of the coordinate system; also the initial area in ft^2 and the coordinates of the sites in ft. The slight difference in height of the three sites is neglected and the initial triangle is assumed flat for the divergence and vorticity computations. The error caused by this assumption is of the order of a fraction of 1 per cent.

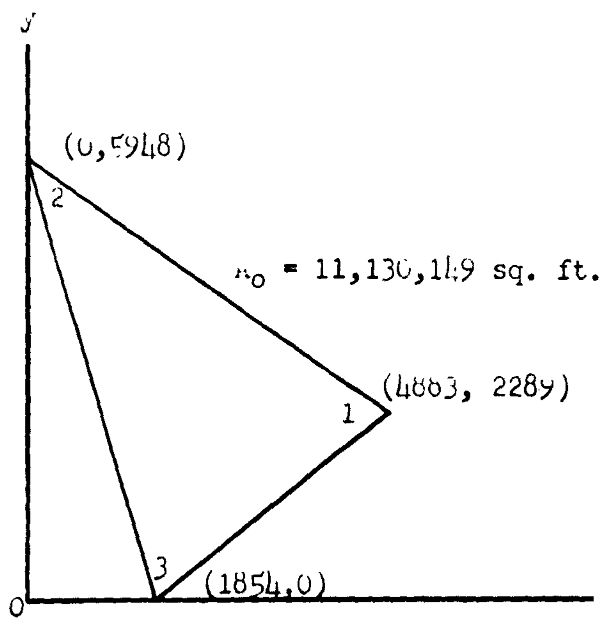


Fig. 1. Coordinates of the three balloons at release time.

The coordinates of the balloons, in feet, for each minute after release were determined by

$$\begin{aligned} (x_n)_t &= (x_n)_o - (R_n \sin \alpha_n)_t \\ (y_n)_t &= (y_n)_o - (R_n \cos \alpha_n)_t \end{aligned} \qquad (3)$$

where α is the usual azimuth reading of the theodolite from north; R, the horizontal range to a point on the surface directly below the balloon, is given by $R_n = z_n \cot e_n$ in which z is the height of the balloon above the surface and e is the elevation angle; and where the subscript t is the time of reading after release, in minutes, the subscript zero refers to the release time, and the subscript n is equal to 1, 2, or 3, depending upon which release site is being examined.

With the area inside the horizontal triangle formed by the three balloons computed for each minute using (2), equation (1) was rewritten as

$$\nabla_H \circ \mathbf{V}_H = \frac{2}{(A_t + A_{t+1})} \frac{(A_{t+1} - A_t)}{60} \qquad (4)$$

where t is again the time of reading of the theodolites and with t = 0 at release time. The value of the $\nabla_H \circ \mathbf{V}_H$, in sec^{-1}, from (4) then represents the mean divergence between levels determined by t and (t+1).

The vertical component of the relative vorticity is given by

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \qquad (5)$$

We can define a vector $\mathbf{W}$ (fig. 2) which is equal in magnitude to the velocity vector $\mathbf{V}$ but is directed 90° to the right of $\mathbf{V}$ [2]. The two-dimensional divergence of this vector is then,

$$\nabla_H \circ \mathbf{W}_H = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \zeta \qquad (6)$$

The coordinates at the time t of the fictitious position of the balloon n (where n is, as before, 1, 2, or 3, depending on the release site under consideration) which moves subject to the "wind" $\mathbf{W}_H$ from observation site n are given by

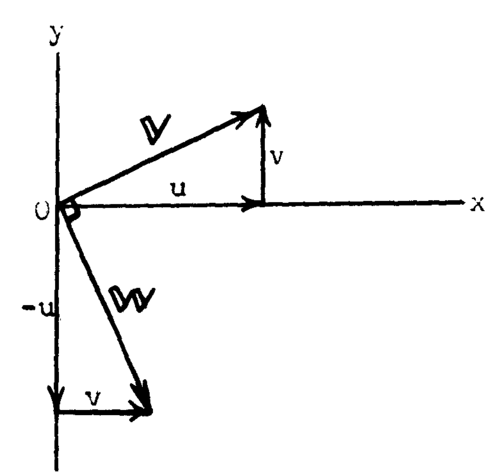


Fig. 2. Definition of vector

$$x'_{no} = x_{no} \quad , \quad y'_{no} = y_{no}$$

and

$$(x'_n)_t = (x'_n)_{t-1} + \Delta y]_{t-1}^{t}$$
$$(y'_n)_t = (y'_n)_{t-1} - \Delta x]_{t-1}^{t} \qquad (7)$$

for t = 1, 2, 3, ... Note that at release time the initial coordinates of $\mathbf{V}_H$ and $\mathbf{W}_H$ at sites 1, 2, and 3 are identical. Therefore, in equations 7, the subscript t cannot be equal to zero, so that in these equations t represents the time of the reading, in minutes, after release. Δx and Δy refer to the differences from one reading to the next of the coordinates of vector $\mathbf{V}_H$ as determined by equations 3.

The vorticity of $\mathbf{V}_H$ is exactly the divergence of $\mathbf{W}_H$ and so is given by relations similar to 2 and 4

$$A'_t = 1/2 \begin{vmatrix} x'_1 & y'_1 & 1 \\ x'_2 & y'_2 & 1 \\ x'_3 & y'_3 & 1 \end{vmatrix} \qquad (8)$$

$$\zeta = \frac{2}{(A'_{t+1} + A'_t)} \, \frac{(A'_{t+1} - A'_t)}{60} \qquad (9)$$

where all quantities have a like meaning, except that the primed quantities refer to vector $\mathbf{W}_H$, the rotated vector.

3. Sample calculation

The method used to determine the divergence is perhaps best shown by an actual example. The three minute ascension of 8 June 1954 (1500 Z) was chosen for this purpose.

Table 1 shows a sample work sheet for the computation of the x and y coordinates for the balloons released at sites 1, 2, and 3. Data in parentheses in this table are the raw data as recorded by the observing teams.

Table 1. Sample worksheet for the computation of the x and y coordinates of the three balloons at the end of each minute interval. (Release: 1000 EST, 8 June 1954.)

Site 1

Minute	(e_1) cot e_1	R_1 [feet]	(α_1) sin α_1	x_1 [feet]	(α_1) cos α_1	y_1 [feet]
1	(32.48) 1.5707	1093	(297.56) -0.88647	5852	(297.56) 0.46278	1783
2	(36.31) 1.3605	1846	(304.00) -0.82904	6413	(304.00) 0.55919	1257
3	(34.82) 1.4379	2859	(308.55) -0.78208	7119	(308.55) 0.62320	507

Site 2

Minute	(e_2) cot e_2	R_2 [feet]	(α_2) sin α_2	x_2 [feet]	(α_2) cos α_2	y_2 [feet]
1	(52.16) 0.77661	541	(316.31) -0.69067	374	(316.31) 0.72317	5557
2	(51.83) 0.78596	1067	(312.38) -0.73865	788	(312.38) 0.67409	5229
3	(49.29) 0.86064	1711	(313.80) -0.72176	1235	(313.80) 0.69214	4764

Site 3

Minute	(e_3) cot e_3	R_3 [feet]	(α_3) sin α_3	x_3 [feet]	(α_3) cos α_3	y_3 [feet]
1	(53.32) 0.74492	518	(302.35) -0.84480	2292	(302.35) 0.53509	-277
2	(54.91) 0.70238	953	(298.75) -0.87673	2690	(298.75) 0.48099	-458
3	(49.35) 0.85862	1707	(309.62) -0.77033	3169	(309.62) 0.63765	-1088

Table 2. Divergence computation worksheet. (Release: 1000 EST, 8 June 1954.)

Coordinates (feet)	Time (minutes) 0		1		2		3
x_1 (Δx_1)	4883	(969)	5852	(561)	6413	(706)	7119
x_2 (Δx_2)	0	(374)	374	(414)	788	(447)	1235
x_3 (Δx_3)	1854	(436)	2292	(398)	2690	(479)	3169
y_1 (Δy_1)	2289	(-506)	1783	(-526)	1257	(-750)	507
y_2 (Δy_2)	5948	(-291)	5557	(-328)	5229	(-465)	4764
y_3 (Δy_3)	0	(-277)	-277	(-181)	-458	(-630)	-1088
Area (sq. ft.)	11,130,149		12,360,060		12,217,316		13,100,065
Divergence ($\times 10^{-5}$ sec^{-1})		174.53		-19.36		116.22	

The coordinates of the balloons released at sites 1, 2, and 3 are recorded again in table 2. The quantities in parentheses in this table are the differences between the coordinates from one-minute reading to the next as indicated in the left hand column. The lower part of the table gives the area at each minute, from equation 2, and the mean divergence between each minute reading from equation 4.

Table 3, which is similar to table 2, shows the coordinates of the three rotated vectors of sites 1, 2, and 3 as computed from the values in parentheses in table 2 and equations 7. The areas of the fictitious

Table 3. Vorticity computation worksheet. (Release: 1000 EST, 8 June 1954.)

Coordinates (feet)	Time (minutes) 0	1	2	3
x'_1	4883	4377	3851	3101
x'_2	0	-391	-719	-1184
x'_3	1854	1577	1396	760
y'_1	2289	1320	759	52
y'_2	5948	5574	5160	4713
y'_3	0	-428	-836	-1315
Area (sq. ft.)	11,130,149	10,146,670	9,046,803	8,371,490
Vorticity ($x10^{-5}$ sec^{-1})	-154.08	-191.01	-129.23	

triangles formed by the rotation of the three horizontal range vectors at each minute reading, from equation 8, are shown in the bottom portion of the table, along with the vorticity values computed from equation 9.

4. The observations

As in the study of the ascentional rates [1], the three balloons for each run were filled in the Weather Bureau helium shelter. In this case, however, it was not possible to check for gas leakage as carefully as for the ascensional rate check releases, but again the balloons were filled to the correct free lift as carefully as possible with the standard equipment. Simultaneous release of the three balloons was accomplished as before by stopwatches which were preset before taking the balloons and equipment to the release sites. Again, all balloon ascensions were made by two-man observation teams reading both the azimuth and elevation angles to 0.01 degrees.

Altogether fourteen successful triple-balloon runs were accomplished, seven day runs and seven night runs. The day runs averaged about 28,000 ft (excluding the three-minute run of June 8) and the night runs averaged about 16,000 ft.

The data obtained were set up on punched cards for machine computation of the divergence and vorticity. Any of the machine computed results which appeared to be out of line were checked by hand and corrected wheneven necessary. It is felt that the resulting values of divergence and vorticity presented herein are free of computational error to a high degree. There is one possible exception, and this particular run (on June 6 at 1500 Z) will be discussed in some detail in section 6. It has been omitted from the divergence and vorticity mean values discussed below.

5. Analysis of the data

All of the divergence and vorticity values obtained are based on one-minute changes in area and orientation of a small triangle (about one mile on a side) as this triangle moved through a vertical distance of about 620 ft. The detail of the small scale wind field was such that some smoothing of the data was desirable in order to see the main features revealed by the study.

a. The divergence

The divergence computations for each run were smoothed by taking five-minute means in the vertical which had a two-minute overlap. That is, a mean value was obtained at three-minute intervals (3, 6, 9, etc) where the sixth-minute value was an average of the fourth through eighth minutes, etc. Then the averages at each three-minute interval for all day runs, all night runs and all runs (day and night) were computed.

Mean curves of day runs, night runs and all runs versus height are shown in fig. 3. It is interesting to note that the 0300 Z and 1500 Z curves are quite similar to about 15,000 ft which is about the extent of the 0300 Z data. The last plotted point on this curve (twenty-seventh minute) is a mean of only three runs while the point just below (twenty-fourth minute) is an average of five runs. Also, the last four points on the 1500 Z curve are means of no more than three or two runs while the fourty-second minute point is a mean of five runs. Both curves are dashed in the region where a sample smaller than five runs is involved. In the 0300 Z curve there is a decrease in divergence with height reaching a minimum at about 6000 ft, then increasing to a maximum just below 12,000 ft, showing a secondary minimum at about 13,500 ft and then increasing again (the top point must be considered doubtful). The 1500 Z curve shows the two minima to be about 1500 ft higher than they were found in the 0300 Z curve but the maximum appears again just below 12,000 ft.

At higher levels the 1500 Z curve increases to a maximum at about 22,000 ft, decreases to another minimum at about 26,000 ft (again considering as doubtful the points based on three runs or less) and appears to increase again at 30,000 ft and above.

Returning to the lowest 15,000 ft of the two curves and the overall mean curve, there are some noteworthy similarities and differences when compared with a similar study of larger scale motion [3]. In [3] it was found that the mean divergence over an area of about 2 x 10^6 miles^2 showed some interesting diurnal variations, namely, the divergence at all levels (below 20,000 ft) was greater at 0300 Z than 1500 Z and the levels of

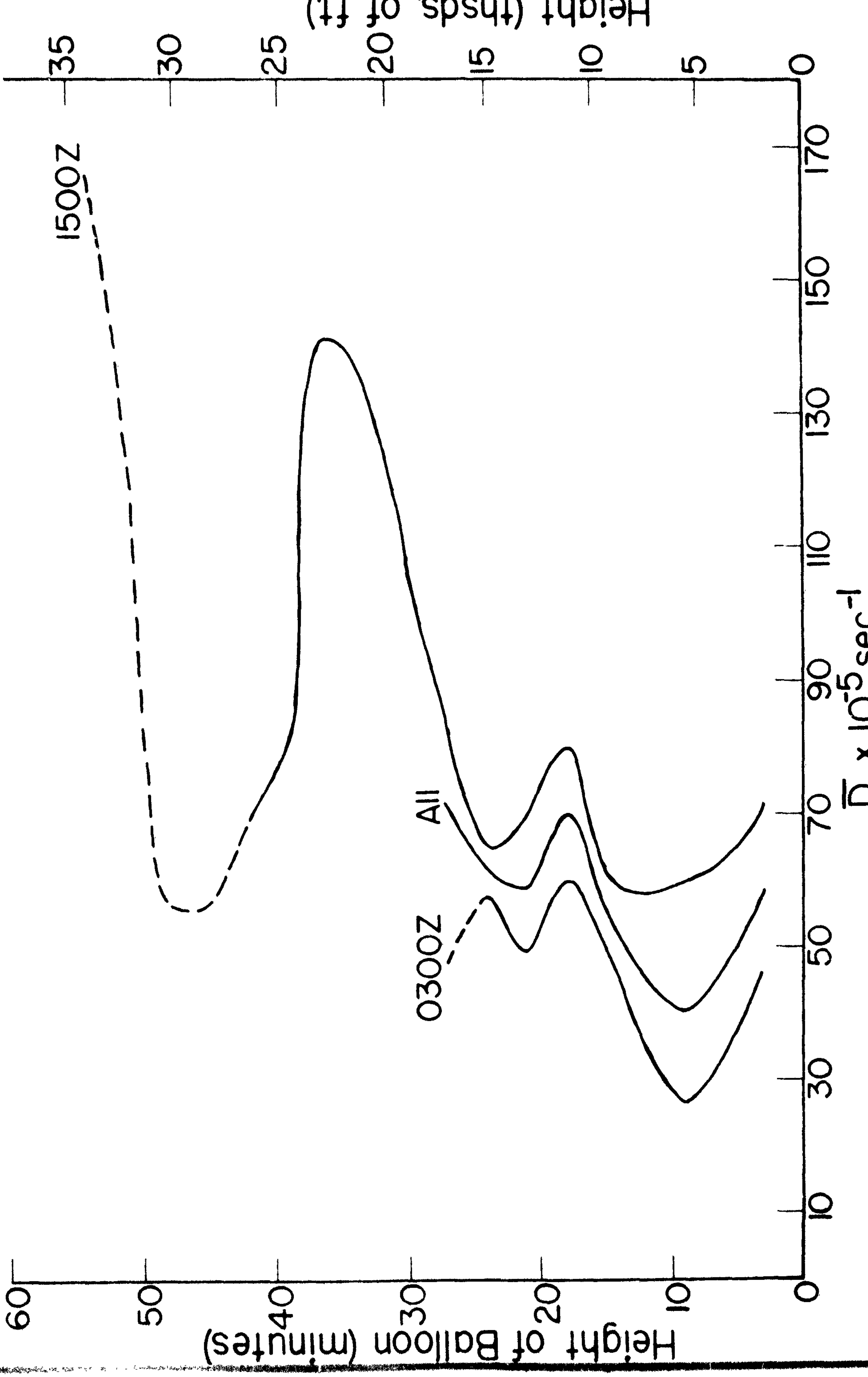
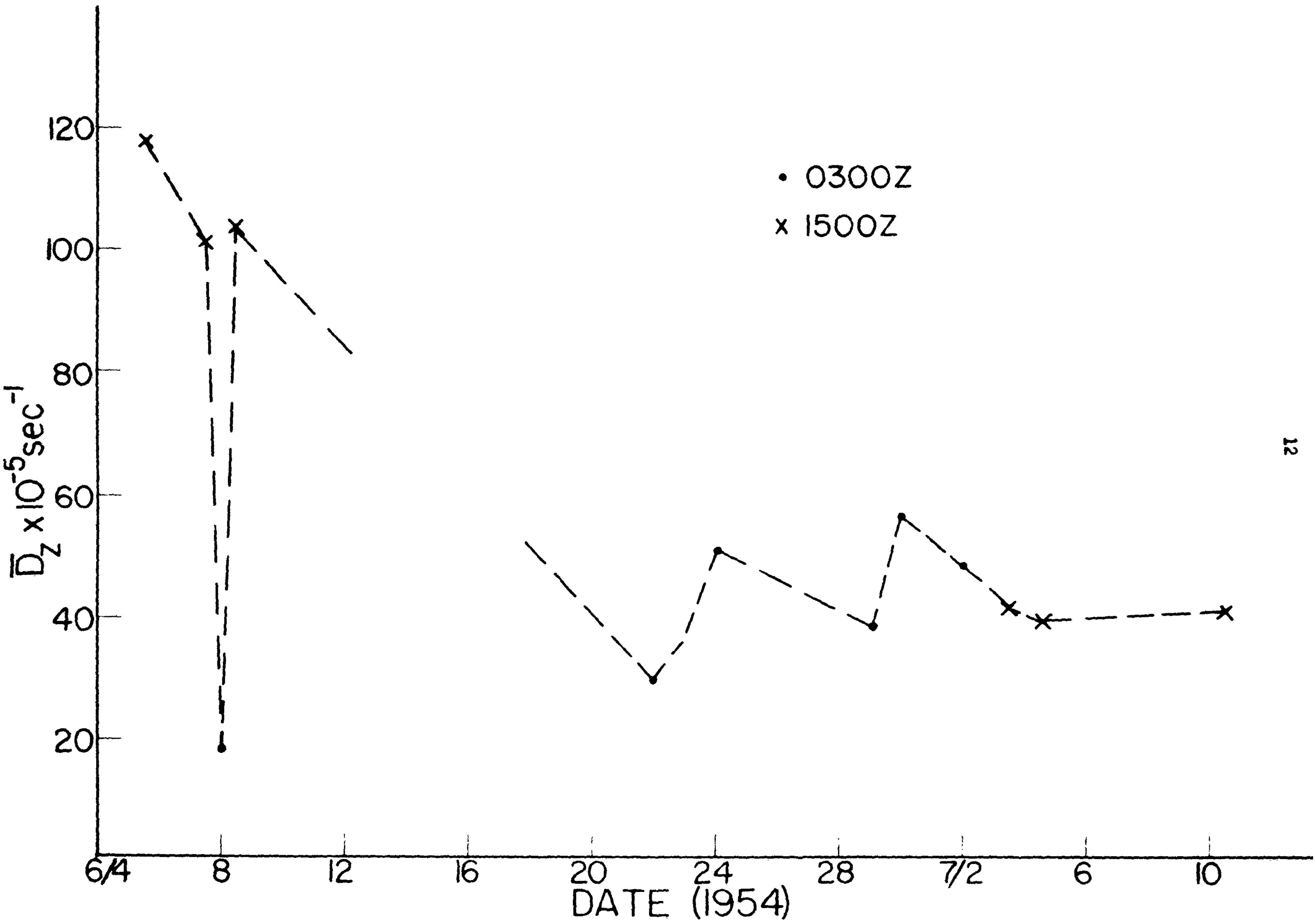

Height (thsds. of ft.)
0
5
10
15
20
25
30
35
1500Z
All
0300Z
Height of Balloon (minutes)
0
10
20
30
40
50
60
10
30
50
70
90
110
130
150
170
$\overline{D}_t \times 10^{-5} sec^{-1}$

Fig. 3

least divergence were lower at 1500 Z than at 0300 Z. The mean 0300 Z and 1500 Z curves in this study seem to be contradictory to the results in [3] in that 0300 Z has smaller divergence values at all levels than 1500 Z and the levels of least divergence are lower at 0300 Z than at 1500 Z. However, the area in the present study ($\sim \frac{1}{2}$ mile^2) is only a point when compared with the area in [3]. There is no reason to suspect that every point in the area considered in [3] would give the same result as the total area. In fact, it would be surprising if such were the case.

The similarities with [3] are that the level of least divergence occurs at 6000 ft in one case and 8000 ft in the other and that a secondary level of minimum divergence occurs at about 13,000 ft in both cases with an apparent increase above this level. It may be quite significant that in two cases which deal with different seasons (spring and summer), different regions (both in size and location) and vastly different scales of motion, that the level where the divergence is noticeably least is in the 6000 - 8000 ft region. This is rather low for a level of non-divergence as it is generally thought of. Also it was shown in [4] that the level of least divergence found in [3] was the level where absolute vorticity was best advected with observed winds.

Fig. 4 may shed some light on the difference in magnitude in the mean divergence at 0300 Z and 1500 Z. This is a curve connecting points of mean divergence for each run averaged over all heights. The mean divergence was apparently larger in early June than in late June and early July, regardless of time of day. The fact that the early June runs were nearly all at 1500 Z may be the sole reason that the mean 1500 Z curve shows larger divergences level for level than the mean 0300 Z curve.



• 0300Z
x 1500Z
120
100
80
60
40
20
$\overline{D}_z \times 10^{-5} sec^{-1}$
6/4
8
12
16
20
24
28
7/2
6
10
DATE (1954)

Fig. 4

Table 4 shows the mean values of divergence for day runs, night runs and all runs. The sign of the divergence was disregarded in obtaining the means. The data in table 4 are confined to the lowest 17,000 ft of the atmosphere. Five of the seven 0300 Z runs went as high as twenty-seven minutes ($\sim$ 17,000 ft) but only three went higher, two of these three terminating at the twenty-ninth minute. Most of the 1500 Z runs went considerably higher but since the 0300 Z and 1500 Z times are averaged together to get an overall value, a common thickness must be used. Therefore, only the first twenty-seven minutes ($\sim$ 17,000 ft) are used in determining the table 4 figures.

Table 4

	1500 Z (day runs)	0300 Z (night runs)	All runs
Number of computations	138	160	298
Mean divergence $\times 10^{-5}$ sec^{-1}	68.9	42.6	54.8

The overall time and height mean divergence value of 54.8×10^{-5} sec^{-1} is a little less than one and one-half orders of magnitude greater than the overall mean value of 1.5×10^{-5} sec^{-1} found for the much larger scale motion in [3].

The above divergence discussion has dealt entirely with smoothed curves and mean values. As mentioned earlier the individual runs, especially when the sign is considered, show much detail of small-scale atmospheric motions. An individual run showing unsmoothed curves of divergence and vorticity versus height is presented in fig. 6 (section 6). It is felt that the many oscillations from plus to minus and vice versa are factual (containing, of course, a percentage of error which will be discussed below). However, we cannot compare these small scale oscillations directly with other studies (such as [3]) or with our synoptic maps due to the vast difference in scale of the observational networks involved. Therefore, the small oscillations were deliberately smoothed out so that such comparisons could be made. Some details of fig. 6 will be discussed in section 6.

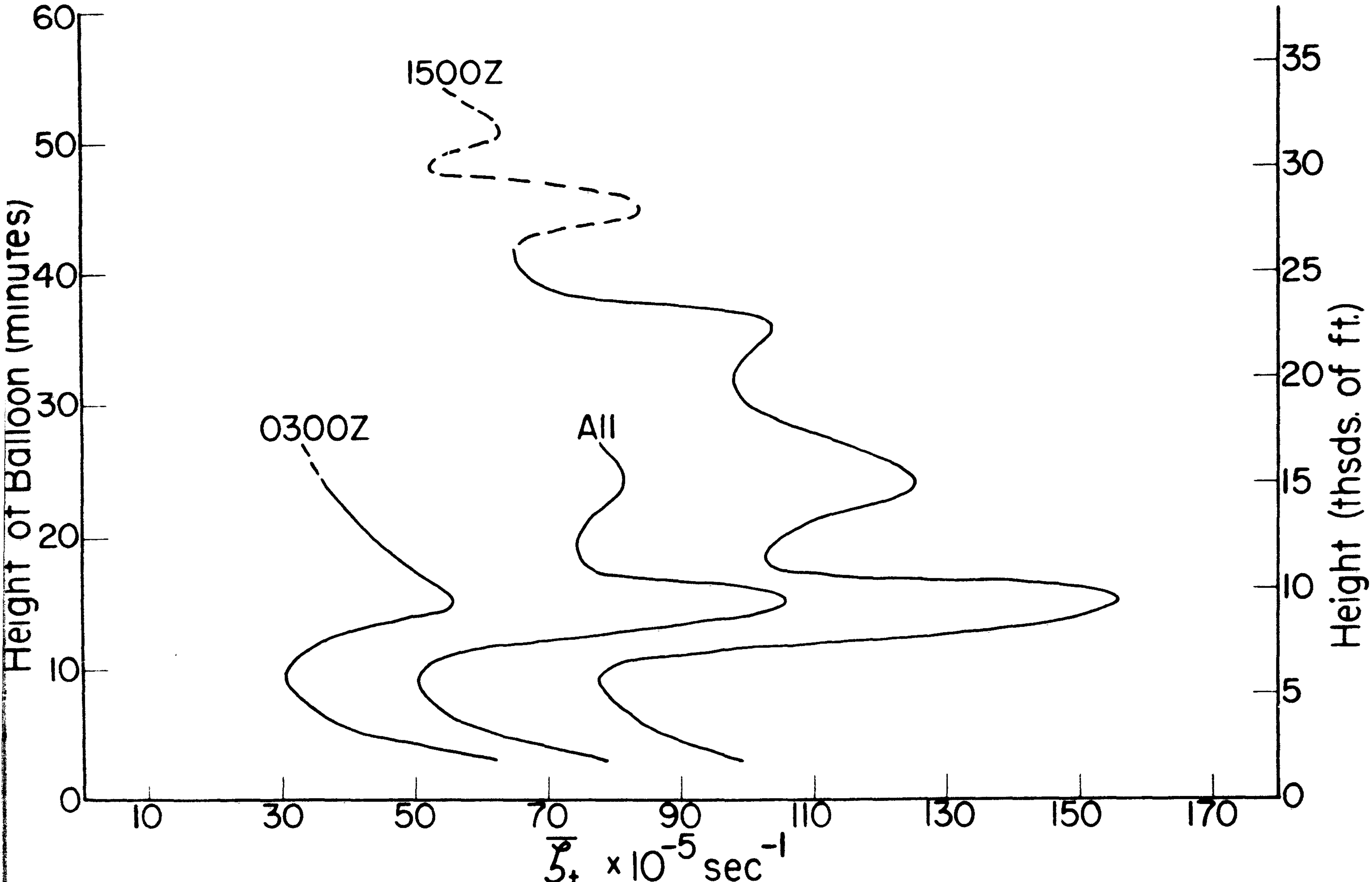
b. The vorticity

The same smoothing technique was applied to the vorticity as was mentioned in the divergence discussion above. Fig. 5 shows the mean curves for 0300 Z, 1500 Z and all runs averaged together. As in the divergence, the 0300 Z vorticity is less at all levels than the 1500 Z vorticity. Also the three curves show quite a bit of similarity to the divergence curves, having maxima and minima at about the same levels up to 17,000 ft.

Table 5 shows mean vorticity values for day runs, night runs and all runs. The overall time and height mean of vorticity = 74.4×10^{-5} sec^{-1} is about 1.4 times as great as the similar value for the divergence in

Table 5

	1500 Z day runs	0300 Z night runs	All runs
Number of computations	138	160	298
Mean vorticity $\times 10^{-5}$ sec^{-1}	108.9	44.6	74.4



Height of Balloon (minutes)
Height (thsds. of ft.)
1500Z
0300Z
All
$\overline{\zeta}_t \times 10^{-5} \sec^{-1}$
0
10
20
30
40
50
60
5
15
25
35
70
90
110
130
150
170

Fig. 5

table 4. Again only the first twenty-seven minutes of data are included in the mean values in table 5.

The mean vorticity values for each run showed the same general trend evident in the divergence in fig. 4. That is, the larger mean vorticities occurred early in June with smaller values in late June and early July. Again the fact that the early June runs were nearly all 1500 Z runs could be the principal reason for the apparent diurnal difference seen in fig. 5. Also the vorticity shows somewhat greater percentual differences between the 0300 Z and 1500 Z mean curves (fig. 5) than does the divergence (fig. 3).

Although the mean vorticity and divergence curves in figs. 5 and 3 appear to indicate a strong correspondence in maximum and minimum values as far as elevation is concerned, the individual runs did not show, minute for minute, any recognizable correspondence of this nature even when absolute values were compared. So that the possible implication that the correspondence of mean values might be inherent in the computing method is not valid in this case.

6. The triple-balloon run of June 6, 1954 at 1500 Z

The June 6th run showed some unusually large divergence values (as large as $\pm$ 2 x 10^{-2} sec^{-1}) and was therefore excluded from the averages shown in table 4 and fig. 3. A more detailed investigation of this run serves to answer some questions concerning it but fails to prove conclusively whether these large divergence values are, or are not, real.

The very large divergence values occurred in the layer from the twenty-second to thirty-second minutes of ascent (approximately 13,000 to 19,000 ft). The curves of divergence and vorticity versus height are shown in fig. 6. It should be mentioned that the assumed rate of ascent,

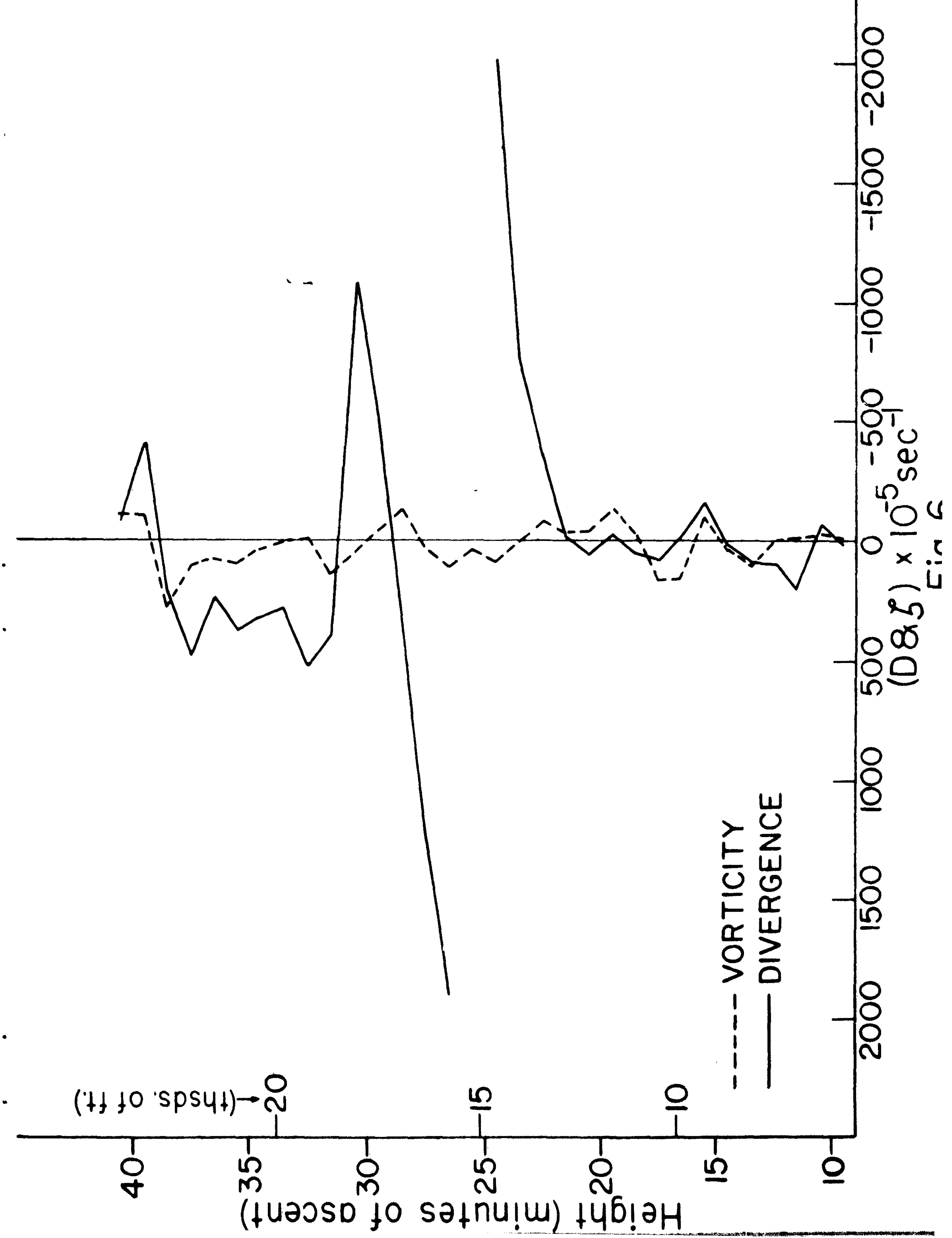

Height (minutes of ascent)
40
35
30
25
20
15
10
(thsds. of ft.) 20
15
10
VORTICITY
DIVERGENCE
2000
1500
1000
500
0
-500
-1000
-1500
-2000
(D & ζ) × 10^-5 sec^-1

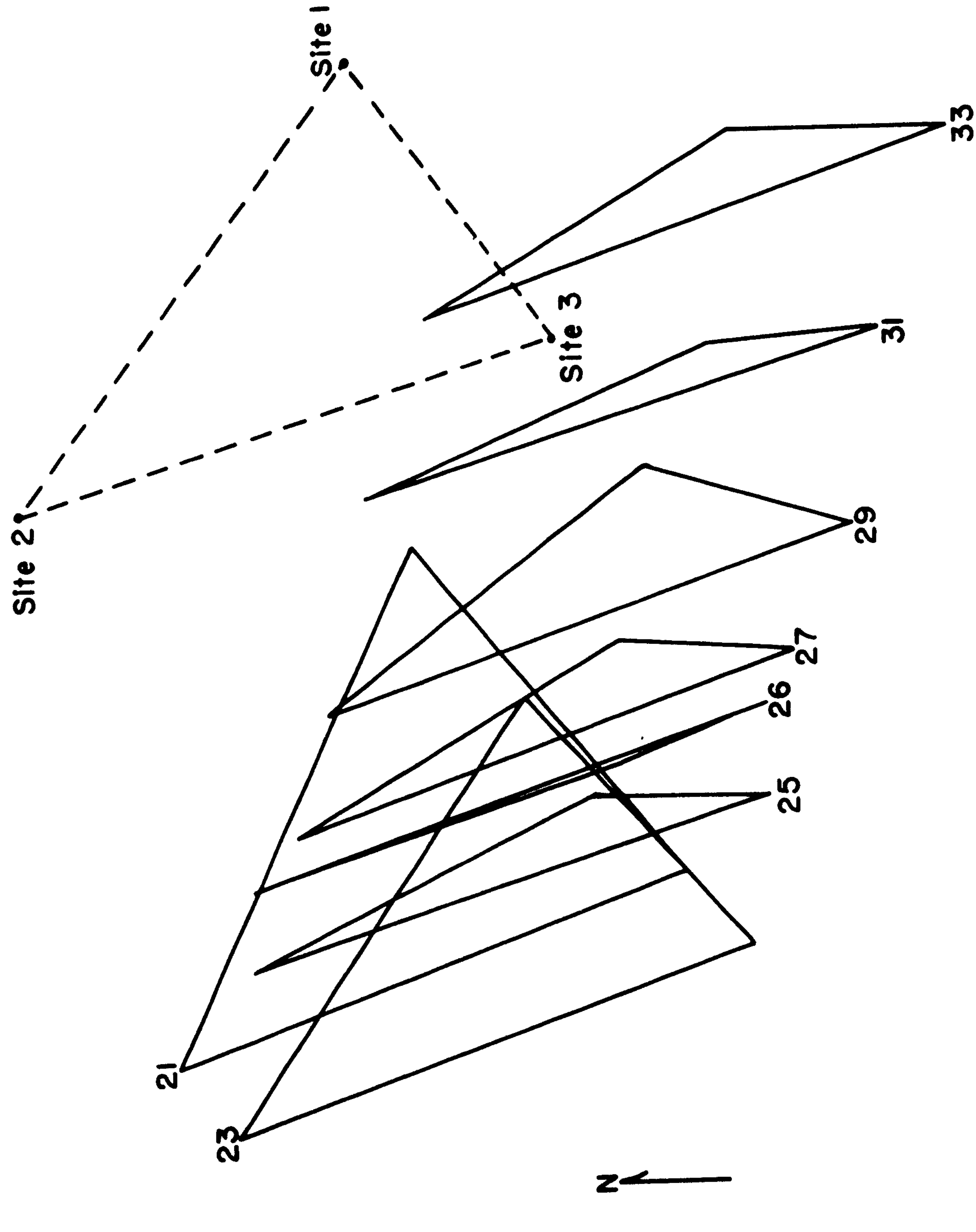
Fig. 6

rather than the day rate determined in [1] was used in working up the three runs so that direct comparison with winds at other stations would be possible. The D and ζ values however are based on a rate [1] which placed the balloons about 600 ft (or one minute) higher in the twenty-second to thirty-second minute region. It will be noted that the vorticity values in the 13,000 to 19,000 ft layer are of the same magnitude as at all other levels. This is because the triangle formed by the balloons was experiencing very little rotation but its area was changing very rapidly.

When this run was taken, the balloon at site 2 was erroneously released two minutes before the prearranged release time. Immediately recognizing the mistake, the observing team decided to follow the site 1 balloon to get a rate of ascent check (see [1]). Later, realizing that the two-minute difference could be adjusted, they switched to the site 2 balloon at the seventh minute and recorded it as the ninth minute since this balloon had been released two minutes early. Although the height of the site 2 balloon run was corrected for the time difference, it might be suspected that this difference in time was instrumental in causing the large divergences which were computed. Examination of the wind direction versus height reveals, however, that sites 2 and 3 are almost identical, particularly from the seventeenth minute (10,000 ft) upward (see fig. 10). It was the site 1 ascent which differed from the other two ascents in the layer of large divergence values. A pronounced change in wind direction (from ENE to W through N) took place in the site 2 and 3 data between the twenty-second and twenty-fourth minutes, but this same change did not take place in the site 1 data until the twenty-fourth to twenty-sixth minutes. The wind speeds versus height bear out the direction curves, with the speed minimum at each site occurring at the same height as the sharp change in wind direction.

The effect of the rapid wind direction change acting two minutes later at site 1 than at sites 2 and 3 is illustrated in fig. 7. The position and orientation of the triangle is shown at time zero (dashed triangle) and at various minute readings during the ascent. Throughout the layer shown (twenty-one to thirty-three minutes of ascent), the side of the triangle connecting sites 2 and 3 remained almost exactly parallel with its original orientation. The triangle of balloons was moving toward the WSW until the twenty-third minute at which time the site 2 and 3 balloons started moving toward the ESE. The site 1 balloon, however, continued to move WSW until the twenty-fifth minute and this caused the area of the triangle to decrease rapidly, actually going to zero just before the twenty-sixth minute reading. The area shows an increase again to the twenty-ninth minute, a decrease from the twenty-ninth to thirty-first minutes and then another increase. The divergence curve in fig. 6 has been broken to indicate that theoretically it goes to and returns from infinity as the triangular area goes to zero. It can be seen that the amount of turning of the triangle was small, therefore the vorticity values in the layer were quite small, averaging only 54×10^{-5} sec^{-1}.

There are at least two possible explanations of the observed results; a) the site 1 balloon could for some reason be lower than the other two by about 1200 ft, and therefore not experiencing the direction change at the same time, or b) the site 1 balloon could be actually experiencing the wind direction change at a higher level than the other two balloons. In connection with a), the idea immediately occurs that the site 1 balloon had a pinhole and was leaking gas. However, all balloons were checked for pinholes when inflated. Also after the balloons were inflated a period of at least fifteen minutes passed before they were released, due to the fact


Site 1
Site 2
Site 3
33
31
29
27
26
25
21
23
N

that all balloons were filled at site 1 and then two of them were transported to sites 2 and 3. Even if a pinhole had gone unnoticed, this time lag would have caused sufficient gas loss before release so that the leaky balloon would rise more slowly throughout the entire run with the difference in rates becoming more noticeable at higher levels. In fact, in one of the runs it was felt that exactly this had happened. The direction curves versus height of two sites were nearly identical with the third showing the same features but at different levels. In this case the lag was one minute in the surface to ninth minute layer, two minutes in the tenth to twenty-fourth minute layer and three minutes in the twenty-fifth to thirty-ninth minute layer. However, in the June 6, 1500 Z run there is a remarkable correspondence between the direction curves of the three balloons up to about the twenty-second minute ($\sim$ 13,000 ft), then a two minute difference appears in the direction changes occurring in the twenty-second to thirty-second minute layer ($\sim$ 13,000 to 19,000 ft). Above 19,000 ft the direction changes with height are so slight that no correspondence or contrast can be noted. This would make the possibility of a preexisting pinhole seem unlikely. Also, if a leak had in some way developed at about the twenty-first or twenty-second minute and was large enough to cause a two-minute lag almost immediately, then one would expect the lag to increase rapidly as time went on. But the lag did not increase noticeably as shown by the plotted directions in the layer between the twenty-sixth and thirty-second minutes (see fig. 10).

It hardly seems plausible that the site 1 balloon is some 1200 ft lower than the other two in the layer under discussion due to differential vertical motions of a small scale. There is pretty general agreement among researchers that $\pm$ 10 $\frac{cm}{sec}$ is a large vertical velocity value, and this is only 3 per cent of the rate of ascent. Even if larger vertical velocities are assumed for smaller scale motions such as may be measured here, it is still difficult to visualize one balloon falling 1200 ft behind the other two in a period of about one or two minutes due to differential vertical velocities. The only clouds present during this ascent were scattered cirrus at about 30,000 ft.

Now, there is also the possibility (b) that the pronounced change in wind direction actually occurred at a higher level at site 1 than at sites 2 and 3. It is largely a product of our thinking in terms of the existing upper wind network that we would accept as normal a 130° change of wind direction in the vertical occurring say at 14,000 ft at one station and at 16,000 ft at a station 100 to 200 miles distant, but would be quite reluctant to accept the idea that this change might occur at 14,000 ft at one station and at 15,000 ft at another if the stations were only one mile apart. There is no reason why this might not happen but we are not accustomed to thinking that it does because we do not normally see it taking place.

A look at the upper wind pattern over the extreme southeastern United States at this time revealed that an anticyclone existed at all levels up to 18,000 ft in southern Alabama. Detailed isogon-isovel analyses of the levels 12,000, 14,000, 16,000 and 18,000 ft indicated that the anticyclone was displaced slightly westward at each succeeding level (see fig. 8). The companion hyperbolic point was found south of Jacksonville at 12,000 ft and was displaced westward to Pensacola at 18,000 ft. Fig. 8 shows the positions of the anticyclone (A) and the hyperbolic point (H) at the levels mentioned above. At 20,000 ft the point singularities could not be found, suggesting that the analysis at that level should show a ridge open to the south. This is reasonable when one considers the trend of (A) and (H) with height shown in fig. 8.

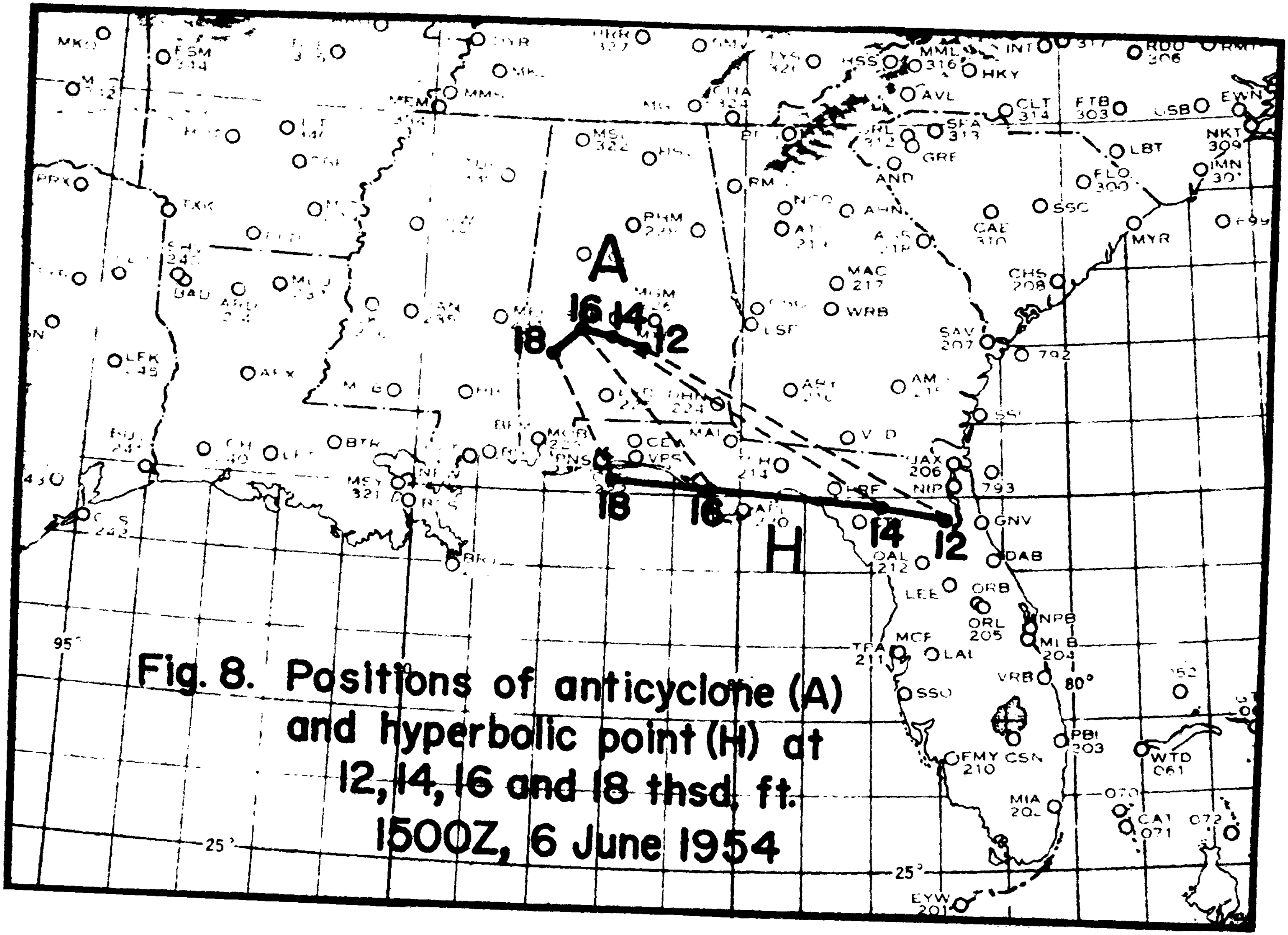


Fig. 8. Positions of anticyclone (A) and hyperbolic point (H) at 12, 14, 16 and 18 thsd. ft. 1500Z, 6 June 1954

Streamlines drawn from ruled isogons and from plotted wind vectors both emphasized the elongated or elliptical shape of this anticyclone, the major axis running WNW-ESE. A freehand streamline analysis at 14,000 ft is shown in fig. 9. Here the elongated shape of the anticyclone has been emphasized more than in the customary analysis which is always smoothed to a scale consistent with the distance between wind reports. That is, instead of assuming that the wind direction changes in a regular fashion from the WNW reports at Maxwell AFB, Birmingham and Atlanta to NE at Tallahassee (some 200 to 250 miles away), it has been assumed that the WNW flow continues to within a short distance of Tallahassee. If this short distance is taken to be about one-half mile, then the directions observed at 14,000 ft at the theodolite sites 2 and 3 are consistent with the larger scale pattern. The direction change had nearly all taken place at sites 2 and 3 by the time the balloons had reached 14,000 ft. The site 1 balloon direction change was completed just above 15,000 ft so that it still showed a northeasterly wind at 14,000 ft. It can be seen in fig. 8 that the hyperbolic point was about at the longitude of Tallahassee at 15,000 ft. As soon as this point assumes a position west of the station (TLH) the station wind must begin to have a westerly component. Although the westward motion of the hyperbolic point can explain the shift in wind direction from ENE to WNW at the station (station here refers to site 1, the official Weather Bureau wind), it does not, or course, explain the direction shift at sites 2 and 3 which occurred mostly below 14,000 ft. However, if one considers that a sharp line of shear may exist between northwesterlies and northeasterlies and that this line lies between site 1 and sites 2 and 3 at 14,000 ft, then it is simple to show (fig. 10) how the observed distribution of three local northeast winds at 12,000 ft, two

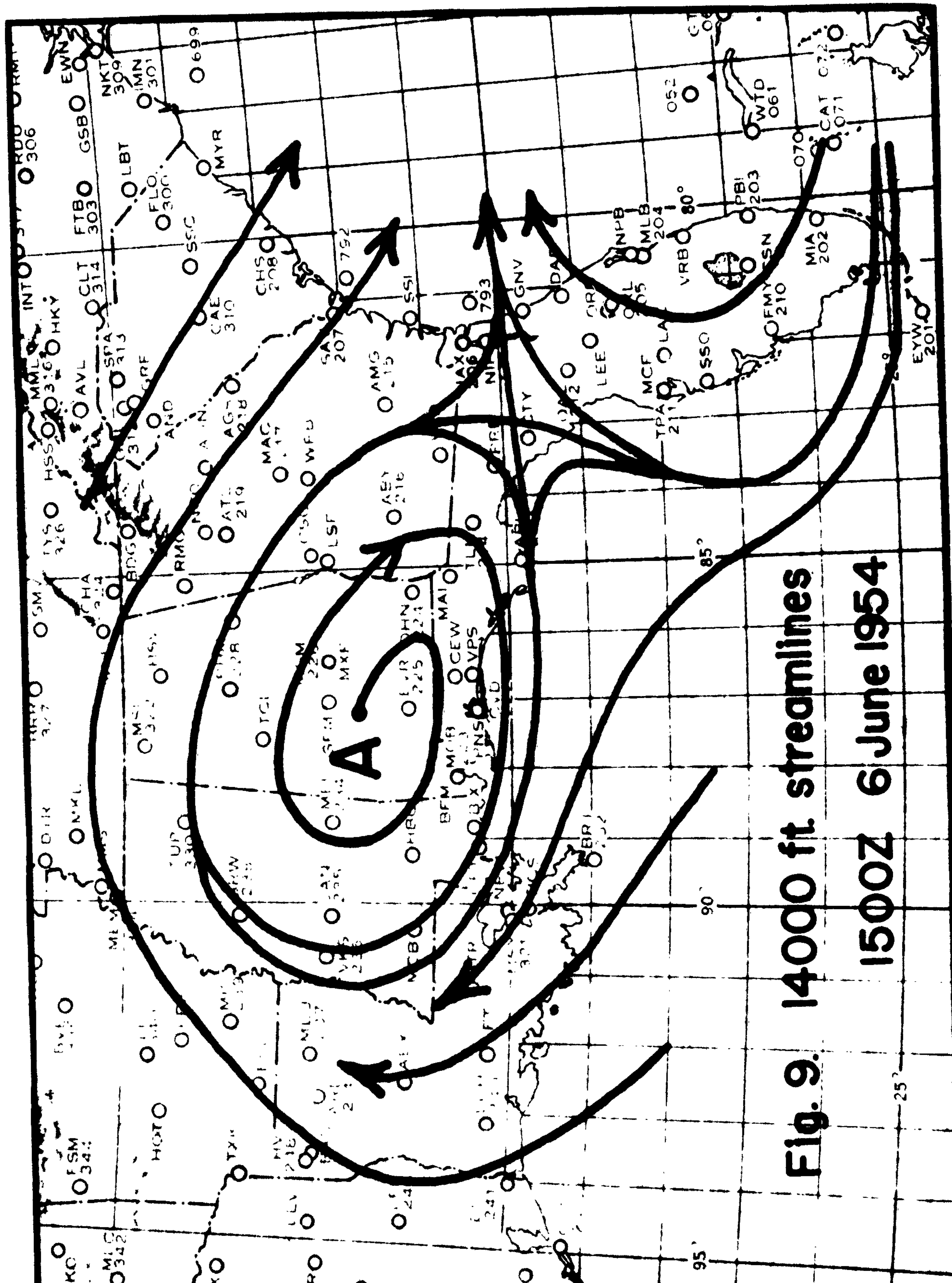


Fig. 9. 14000 ft streamlines 1500Z 6 June 1954

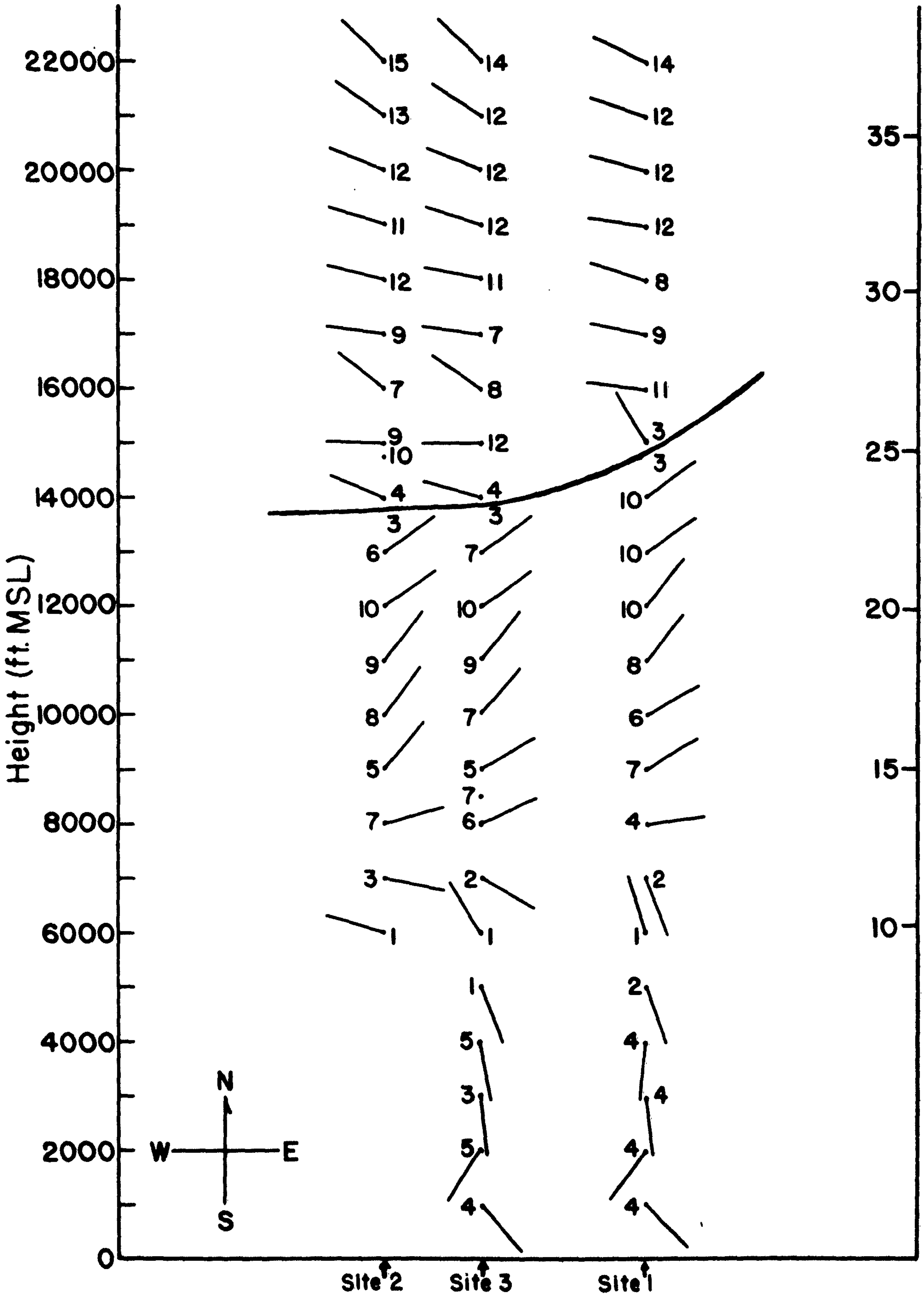


Fig.10 Wind direction (°) and speed (knots) at 1500Z, 6 June 195

northwest winds and one northeast wind at 14,000, and three local northwest winds at 16,000 ft and higher could occur. The heavy line in fig. 10 shows the separation of easterly from westerly winds. We are accustomed to thinking of this heavy line as nearly horizontal over such small linear distances as 3029 ft, which is the east-west distance between sites 3 and 1. In fig. 10, where there is a one to one ratio between ordinate and abscissa, a maximum tilt of 21° from the horizontal would be required for the heavy line and this maximum would occur between sites 3 and 1.

This argument cannot be definitely proved nor disproved from the data available. It is an entirely possible explanation which allows the data from the three sites to be consistent locally and consistent with the larger scale pattern. Those working with wind fields on a small scale are continually finding evidence that such things may be more common than generally thought. McLean and Rados [5] recently noted a very sharp shear-line at 30,000 ft. It was noted from aircraft observed winds and could not be found with the coverage of conventional winds. This shear-line (located about 100 miles east of Tallahassee in April 1954) showed a 180° wind direction change in a distance of about 20 miles (the distance between observations) with speeds of about 30 knots on each side of the line of direction change. This shear-line might have shown up as even sharper if observations had been available at smaller distance intervals. The McLean-Rados shear-line was found on each of two successive passes by the aircraft through the region at times about two hours apart. Except for minor details this shear-line fit into a conventional contour analysis at the same level and time; however much additional information was given by the aircraft observations.

7. Discussion of errors

An attempt has been made to determine as nearly as possible the percentual error included in the computed divergence and vorticity values.

The three largest sources of error are:

1) Whatever difference exists in rates of ascent of three balloons released simultaneously in a small region, 2) the difference between the rate of ascent on any given day or night and the mean ascent rates used for the computations and 3) the errors in the azimuth angle readings - particularly when the flight of the balloon causes rapid time changes in azimuth.

Error 1: The error caused by differences in ascent rates of three balloons released simultaneously is not available from the data gathered. However such differences as do exist would be due to very small scale atmospheric vertical motions, since the balloons were released in a small region, and should be quite random.

A spot check was made of divergence values of (10, 50, 100, 150, 200, 300) x 10^{-5} sec^{-1} by introducing an assumed error in heights. At one minute the heights originally used were changed by (+10, 0, -10) ft and heights at the next minute were left unchanged. This amounts to including a 20 ft error between stations. The divergence values were recomputed and the percentual errors are shown in table 6. Two such spot checks were made for each divergence value listed above and the check points included nearly all of the triple-balloon runs.

Error 2: It was found that the individual runs in Part 1 differed from the mean rates of ascent by 18 ft/min on the average, or a percentual difference of $<$ 3 per cent of the mean ascent rate. It is difficult to tell how much the computations are in error due to this effect. It is probable that, on a given day or night, all three balloons will rise faster or slower or the same as the mean rate used in the computations. It has been assumed that the one minute thicknesses are 18 ft in error due to this effect and the same percentual figures (see table 6) have been used that were applied in the case of error 1 where a 20 ft error was assumed.

Error 3: At the same check points used in determining error 1, azimuth angle reading errors of (+.02, 0, -.02) degrees were introduced into the original angles at the first minute and the readings at the next minute were left unchanged. The divergence values were recomputed and the results are shown in table 6.

Error 4: This error has been added to the table to take care of the various very small errors such as:

a. A slight error which is introduced due to human inability to exactly orient the three theodolites, thereby causing a very slight random tilt in the original triangle.

b. The ground was not exactly flat and this introduces a slight systematic tilt in the original triangle. Both a and b are extremely small and are carried through the entire run. b will be the same for all runs. These two errors are considered negligible.

c. The computing method is such that very small angles in the geometry of the setup can cause large errors. It is felt that this error has been reduced to the negligible class by eliminating all computations which contained trigonometric function values smaller than or equal to 0.10000.

d. Readings of angles may not have been made at exactly the same time at the three installations. In all cases the readings were either on time or late, never early. In all cases of late readings an attempt was made to note the lateness so that the values could be suitably adjusted later when the data were worked up. It is felt that this procedure reduced this error to the negligible class.

e. Only brief mention will be given to errors in reading elevation angles because; 1) this angle changes slowly and can be read more accurately than the azimuth, and 2) any error which might exist here is not independent of rate of ascent differences which were considered under errors 1 and 2.

f. In some cases when a reading was missed at one installation, interpolated readings have been substituted. Examination of the runs indicates that this can be done with sufficient accuracy to justify the practice.

The total of these errors 4a, b, c, d, e, f has been assumed to average 3 per cent of all divergence values.

Table 6

Individual and total percentual errors for various divergence values.

Average Errors (%)	Divergence x 10^{-5} sec^{-1} 10	50	100	150	200
1	23	5	3	1	1
2	23	5	3	1	1
3	8	4	3	$1\frac{1}{2}$	$\frac{1}{2}$
4	3	3	3	3	3
Total	33	8.5	6	3.7	3.4

The total average error in table 6 is the square root of the sum of the squares of the individual average errors. The individual errors are assumed to be independent and random. This total average error is graphed in fig. 11 and clearly shows the dependence on the magnitude of the divergence. The mean value of divergence (for the first twenty-seven minutes) was 54.8×10^{-5} sec^{-1} and this value is about 8 per cent in error. Values smaller than 20×10^{-5} sec^{-1} are $>$20 per cent in error. The vorticity computations can be assumed to have corresponding errors. The mean vorticity of 74.4×10^{-5} sec^{-1} is in error by 8 per cent and values smaller than 30×10^{-5} sec^{-1} are $>$20 per cent in error. A simulated curve for vorticity has been included in fig. 11.

8. Conclusions

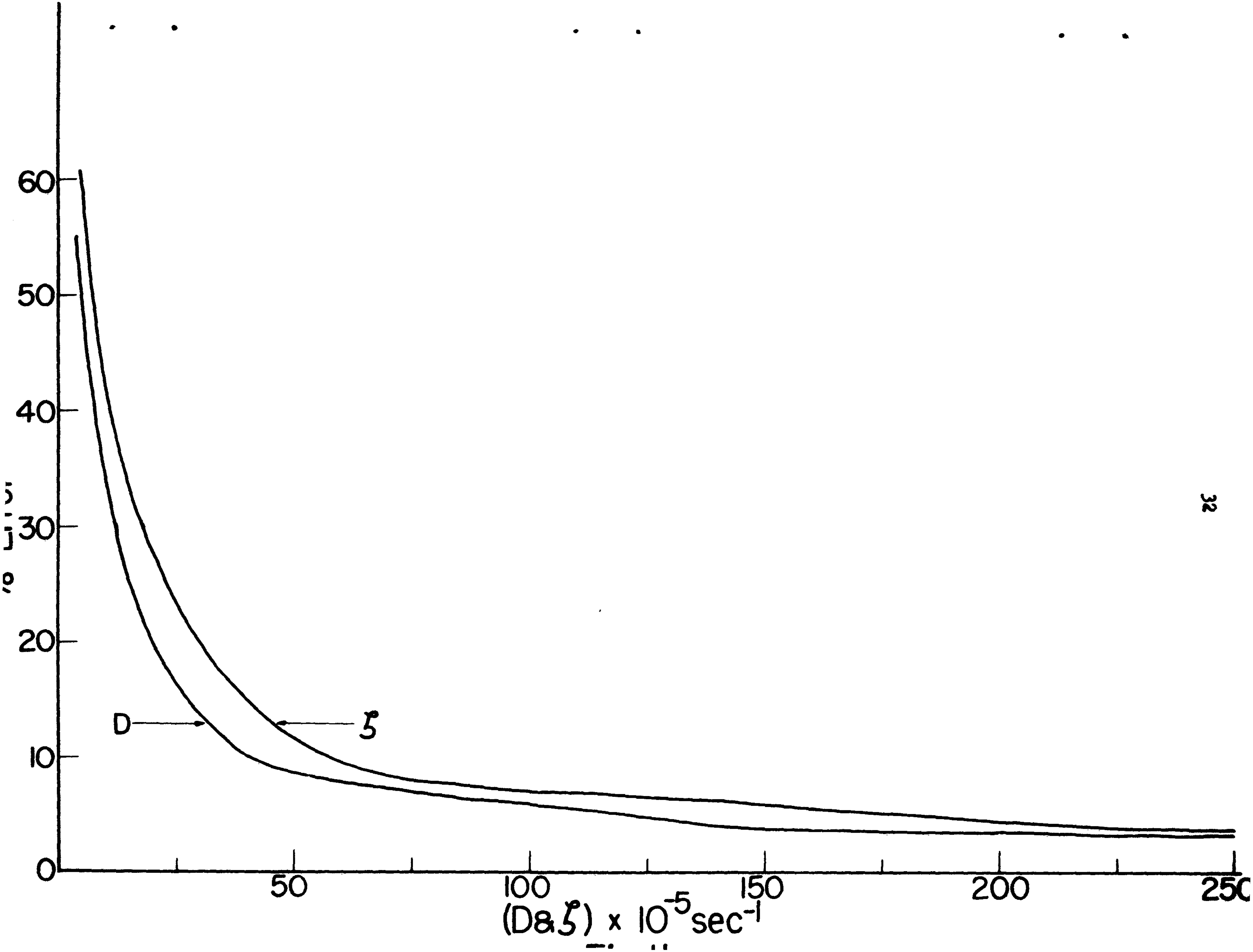
a. The magnitudes of the divergence and vorticity measured over a small area of about $\frac{1}{2}$ square mile were found to be about 55×10^{-5} sec^{-1} and 75×10^{-5} sec^{-1}, respectively.

b. It is felt that the values quoted in (a) are in error not more than 10 per cent.

c. Primary and secondary levels of least divergence were found at 6000 ft and 13,000 ft. In [3] similar levels were found at 8000 ft and 13,000 ft. The study [3] dealt with a different geographical region, a different season and much larger scale motion.

d. The absolute value of the divergence did not appear to vary diurnally (as in [3]), but there is some evidence that it varied somewhat systematically throughout the observational period of about six weeks (see fig. 4).

e. One triple-balloon run showed possible evidence of a very narrow line of wind direction change in the horizontal. Postulating the existence of such a line, the small scale wind observations can be shown to be consistent with one another as well as with the larger scale motion.


60
50
40
30
20
10
0
D
ζ
50
100
150
200
250
(D& ζ) x 10^-5 sec^-1

f. It follows from (e) that large values of divergence ($\pm 10^{-2}$ sec^{-1}) may be found locally (over a $\frac{1}{2}$ square mile area). However, even on this small scale, such large values were found to be the exception rather than the rule.

Acknowledgements -- Many persons assisted in the field work connected with this project. The writer wishes to thank Mr. Jesse W. Smith, Chief, and all of the personnel of the U.S. Weather Bureau station at Tallahassee; also Mr. Lloyd Tourville, Lt. Charles Diaz, U.S.A.F., and other graduate and undergraduate students at Florida State University who participated in this phase of the work. Special thanks are due Capt. Milton H. Sipple Jr., U.S.A.F., who supervised the extensive field operations, checked the data obtained and arranged the data for punched card computations.

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